

Second-order Stark effect for hydrogen line intensities

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The Stark shift and splitting for arbitrary energy levels in hydrogen have been calculated up to very high orders in electric field F . Meanwhile, the intensity of Stark lines of an atom in field was supposed to be field-independent.

The field dependence of the line intensity had been observed first by Bellerman et al for the E1 transitions between highly excited Rydberg states [1]. In their numerical calculations of the field-induced variation for the upper-level wave function they took into account the field-induced mixing for a limited set of nearest states when diagonalizing the energy matrix.

We have recently considered the field dependence of the radiation transitions in the first order of perturbation theory (PT) [2], with the use of the Sturm-series expansion for the Coulomb Green function in parabolic coordinates. This method takes naturally into account the field-induced mixing to the initial and final Stark levels of all atomic eigenstates with different principal quantum numbers n , including continuum.

In a strong-field region the first-order corrections to the line intensities and matrix elements become insufficient and the higher-order corrections are needed for more precise calculations. Moreover, the lines with independent of field intensity in the first order of PT [2] had been discovered. To evaluate the behaviour of intensity for such lines in field the second-order corrections should be calculated. In addition, the second-order corrections will provide information on the range of fields where the PT approach is valid.

We report on the analytical results we have derived for the hydrogen Stark line intensity in dc electric field in the second order of the field strength. The range of fields is considered where the fine-structure effects may be neglected and the parabolic numbers, $n_1 + n_2 + m + 1 = n$ are good integrals of motion, so the PT in the basis of parabolic wave functions is used. However, a significant difference from the first-order PT appears in the second order, since the states of degenerate level with $n'_1 = n_1 \pm 1$, $n'_2 = n_2 \mp 1$ contribute in this order and a consecutive PT **for degenerate states** should be used to derive analytical expressions for the second-order wave functions. The line intensity in field F may be presented as the power series of F :

$$I_{nn'}(F) = I_{nn'}(0) \left(1 + F\beta_{nn'} + F^2\gamma_{nn'} + \dots \right), \quad (1)$$

where $I_{nn'}(0)$ is the intensity in the zero-field limit. The factors for the first- and second-order corrections to intensity in field come from corrections to the two factors of the radiation intensity $I_{nn'} \sim \omega^4 |d_{nn'}|^2$, i.e. the line frequency $\omega_{nn'}$ and the dipole matrix element $d_{nn'}$. They

may be written as follows:

$$\beta_{nn'} = 4 \frac{\omega_{nn'}^{(1)}}{\omega_{nn'}^{(0)}} + 2 \frac{d_{nn'}^{(1)}}{d_{nn'}^{(0)}}, \quad \gamma_{nn'} = \gamma_{nn'}^{(\omega)} + \gamma_{nn'}^{(d)}, \quad (2)$$

$$\text{where} \quad \gamma_{nn'}^{(\omega)} = 4 \frac{\omega_{nn'}^{(2)}}{\omega_{nn'}^{(0)}} + 6 \left(\frac{\omega_{nn'}^{(1)}}{\omega_{nn'}^{(0)}} \right)^2, \quad \gamma_{nn'}^{(d)} = 8 \frac{\omega_{nn'}^{(1)}}{\omega_{nn'}^{(0)}} \cdot \frac{d_{nn'}^{(1)}}{d_{nn'}^{(0)}} + \left(\frac{d_{nn'}^{(1)}}{d_{nn'}^{(0)}} \right)^2 + 2 \frac{d_{nn'}^{(2)}}{d_{nn'}^{(0)}} \quad (3)$$

The term $\gamma_{nn'}^{(\omega)}$ is the contribution of the frequency variation in field, if the dipole matrix element is field-independent, whereas $\gamma_{nn'}^{(d)}$ is due to the field corrections to the matrix element. Numerical computations show that in majority of cases (if $|n - n'| > 2$), $|\gamma_{nn'}^{(d)}|$ exceeds $|\gamma_{nn'}^{(\omega)}|$ both for π - and σ -transitions. For example, $\gamma^{(d)}((30, 26, 0, 3) \xrightarrow{\pi} (4, 0, 0, 3)) = -7.7 \cdot 10^{12}$, $\gamma^{(\omega)}((30, 26, 0, 3) \xrightarrow{\pi} (4, 0, 0, 3)) = -7.8 \cdot 10^{10}$. It means, that the main contribution to the second-order correction to intensity of such lines comes from the dipole matrix element.

If the upper-state quantum number n is large enough, the second-order correction $F^2 \gamma_{nn'}$ becomes equal to the first-order one $F \beta_{nn'}$, when the upper level is already ionized by field. Hence, the second-order PT is valid for determining the dependence on field of the atomic line intensities in all practically interesting field ranges. E.g. for the transition $(10, 9, 0, 0) \rightarrow (1, 0, 0, 0)$ the equality $F^2 \gamma_{nn'} = F \beta_{nn'}$ holds in the field $F = 1.7 \cdot 10^{-5} a.u.$, which exceeds the critical field value for the upper level $F_{cr} \approx 1.2 \cdot 10^{-5} a.u.$ The numerical data we obtained for transitions between Stark components of highly excited Rydberg levels are in a good agreement with the experimental results of ref. [1] for all the field ranges presented therein.

If the final (lower) state is fixed, we can write analytical expressions for the quantities $d_{nn'}^{(2)}/d_{nn'}^{(0)}$ and $\gamma_{nn'}$ as functions of the initial- (upper-)state quantum numbers $n, q = n_1 - n_2, m$. E.g. for Lyman π -transitions

$$\gamma_{(n, n_1, n_2, 0) \xrightarrow{\pi} (1, 0, 0, 0)} = -\frac{1}{48(n^2 - 1)q^2 Z^6} \left[(450n^{10} - 236n^8 - 326n^6 - 8n^4 + 720n^2 - 744)q^2 - 3n^6(n^2 - 1)(3n^2 + 1)^2 + 3n^2(19n^6 - 15n^4 - 48n^2 + 56)q^4 \right], \quad (4)$$

where Z is the nuclear charge of the hydrogenlike atom. A similar equation may be written for the the second-order corrections to intensities of σ -lines. In particular, this case includes the transitions with the first-order corrections equal to zero ($\beta_{n_1}^\sigma = -2qn/Z^3 = 0$ for $q = 0$).

$$\gamma_{(n, n_1, n_2, 1) \xrightarrow{\sigma} (1, 0, 0, 0)} = -\frac{1}{48(n^2 - 1)Z^6} \cdot \left[3n^2(19n^6 - 15n^4 - 48n^2 + 56)q^2 + 543n^{10} - 491n^8 - 176n^6 + 40n^4 + 720n^2 - 744 \right] \quad (5)$$

[1] Bellermann M, Bergeman T, Haffmans A, Koch P M and Sirko L *Phys Rev. A.* **46** 5836-44 (1992).

[2] A A Kamenski and V D Ovsiannikov, *J.Phys. B: At. Mol. Opt. Phys.* **33** 491-505 (2000).